CERME: Numerical cognition with Touchcounts or Rakin? An enactive and ecological approach to finger gnosis

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Our paper aims to provide a proof of concept about the theoretical framework of enactive and ecological approaches to the field of perceptual learning of mathematics with digital technology. We reporst on the finger gnosis or finger knowledge that school children deploy when engaging with digital technologies such as Touchcounts and Rakin. From our theoretical lens we contrast both applications and conclude that they offer rich possibilities for number learning, however, Touchcounts adheres better with enactive and ecological foundations by prompting finger gnosis in different ways, while the Rakin technology application restricts actions and gestures, leaning toward representationalist cognitivism.

Introduction

Digital technology poses significant research challenges in understanding early mathematics learning (Clark-Wilson, Robutti and Sinclair, 2014). Here, we are interested here in discussing theoretical approaches to cognition and mathematical learning that are considered in the design of digital technology and that explore alternative aspects to representationalism. We share with Glenberg (2006) that any attempt at educational technology in mathematics should be oriented to an action-based human cognitive system, rather than considering the cognitive system as if it were a computer. In this article we emphasize how digital technology promotes new ways of learning by doing within the framework of new contemporary approaches to enactive and ecological cognition (Pardo, Rosenbaum and Abrahamson, 2021). This is relevant in the face of the persistence of technological resources in mathematics that still embrace cognitivist hangovers, which disavow the body and the environment as sources of knowledge (Hutto, Kirchhoff and Abrahamson, 2015). The evidence on technology and mathematics education from an embodied, intersubjective and instrumental approach is auspicious, see Nemirovsky et al., 2013; Radford, 2014; Sinclair & de Freitas, 2014; Drijvers, 2019; Abrahamson, 2021). In the framework of these studies, we place special attention to the understanding of cognition and digital learning from a dynamic sensorimotor theory based on the progressive structural coupling between the agent and the sociotechnological environment (Videla, Aguayo, & Veloz, 2021). We highlight and share the contribution of Shvarts et al (2021) on the importance of instrumented action constituted by a body-artefact dynamic functional system that regulates the actions of the agent and the sociomaterial environment. As a complement and from our enactive and ecological theoretical lens, we argue that digital technologies should be presented as ecological niches of skilled sensorimotor expansion, that is to say , they enhance the ecological control of actions for the achievement of learning goals. We highlight the movement of hands and fingers in the framework of perceptual and gestural exploration with tactile technology for the understanding of number. Particularly, we emphasize "finger gnosis", which consists in the knowledge of numbers through finger movements on touch screens. The aim of our study is to contrast the use of Jackiw and Sinclair's TouchCounts (2014) and CEDETI's Rakin (2021) applications for number comprehension, in light of our theoretical approach that highlights the specialization and expansion of actions and gestures that make it possible to bring a world of numbers and motion.

Theoretical framework

Representationalism: cognitive inertia

In this section we give brief insights into the origins and characteristics of representationalism with the aim of contextualizing traditional approaches to cognition. Generally any kind of cognition-driven behavior beyond reflexive reactions in the mediated environment requires computational operations of contending representations (Myin and Zadhari, 2018). Fodor (1983) is one of the forerunners of the representationalist approach based on the modularity of the mind, as he asserts the impossibility of genuine conceptual learning outside the set of pre-existing representations. Representationalism is based on the idea that cognition operates on the basis of representations of intrinsic functionality called internalism or objective representations of a world independent of the subject called externalism (Piccinini, 2004). There is also the idea of computationalism in which cognition is reduced to processes and operations performed on entities that carry some form of content, functionally structured from input and output channels of information (Sprevak, 2010).

Enactivism: cognitive agency

In this section we provide brief clarifications of the role of perception in enactivism in order to illustrate the sub-personal dimension of agency. In face of the persistence of the traditional cognitivist approach based on representationalism, computationalism and internalism, enactivism emphasizes the role of action for perception by pointing out that: "(1) perception consists of perceptually guided action and (2) cognitive structures emerge from recurrent sensorimotor patterns that allow action to be perceptually guided" (Varela, Thompson and Rosch, 1991:173). This implies that cognition does not require mental representations to cause skilled bodily activity, since enactivism is framed within the circular dynamics of the perception-action loop in which cognition is action. When the enactive approach is alluded to, it refutes the idea of cognitive processing and emphasizes the notion of a decentralized cognitive system based on autonomy, sense-making, embodiment, emergence and experience (Di Paolo, 2020). Embodiment refers to movements, gestures and multimodal perception intertwined and constitutive of cognition through experience. Also, enactivism embraces the principles of the biology of knowledge, in which living beings are considered adaptive, autonomous agents and creators of their own worlds as a result of their history of coupling with the environment (Maturana and Varela, 1980). A cognitive system means that interchange with the world are inherently meaningful for the knower, given that movements are at the center of mental activity (Froese, 2011).

Ecological psychology: skillful expansion

Next, the role of perception from ecological psychology is presented in order to understand the interpersonal relationship between agent and sociomaterial environment. J.J. Gibson's (1979) ecological psychology proposes that cognition is enacted, shaped and structured by reciprocal interactions between the organism and the environment. Gibson (1979) argues that the environment is hierarchical in ecological information available to support everyday activities, think of; roads, rocks, slopes, forests, buildings, technology, and actions among others (Heft, 2021). Ecological psychology proposes a relevant epistemological debate by promoting an understanding of the world that overcomes dichotomies: perception/action; organism/environment; subjective/objective and mind/body. These dichotomies are at the basis of the theoretical assumptions of behaviourist and cognitivist psychology, which adscribe to the poverty of the stimulus, the passiveness of perception and the processing of information. Hence the importance of ecological information for Gibson, since it depends on the specification of the relationship established by the legal covariation between energy-optical, mechanical and chemical patterns, when actively participating in the environment (Chemero, 2009). This mode of participation, highlights affordances that are conceived as a relationship between an aspect of the sociomaterial environment and an ability available in a life form (Rietveld and Kiverstein 2014). By life form, we will understand a type of cognitive agent with a certain type of practice within an ecological niche (Rietveld et al. 2013; Bruineberg and Rietveld 2014).

Brief Overview Tactile Technology and Numerical Cognition

In this article, we ascribe to the radically enactive approaches of numerical cognition developed by Zahidi and Myin (2016) on the importance of the body in the understanding of numbers and counting: correspondence, ordinality, and cardinality.

Rankin

Rakin is an inclusive Chilean application that seeks to promote the learning of mathematics in preschool children through a virtual desktop interface where skills such as seriation, classification, conservation of quantities, numbers, counting, cardinality and ordinality can be stimulated. In addition to being aligned with many of Chile's preschool mathematics learning objectives (CEDETI, 2021).

Touchcounts

TouchCounts (Jackiw & Sinclair, 2014) is an app that allows young learners to simultaneously coordinate various forms of numbers: number names such as 'three', number touches on the screen, number of records on the screen, and number symbols such as 3. It represents a multimodal correspondence between touching with fingers, seeing numbers, and hearing number words (a one-to-one correspondence of touch, sight, and sound). The application has two worlds: enumeration and performance.

Our enactive-ecological proposal: learning number with Touchcounts or Rakin?

Our proposal consists of a dynamic approach to sensorimotor agency for understanding the enactive-ecological approach to numerical cognition, see Videla, Aguayo and Veloz (2021). The unification of these approaches seen as a continuum relieves the human cognitive-technological tool assemblage beyond the subpersonal dimension of enactivism and the interpersonal dimension of ecological psychology (Heras-Escribano, 2018). In the case of number learning with digital technology, we propose that a form of human-tool assembly is only possible from the ecological niche. Following Maturana and Dávila (2015), "the ecological niche of an organism does not exist outside the ecological niche that makes it possible, since the organism and its niche are reciprocally constituted inseparably" (p. 159). In this sense, the ecological niche is the meta-relationship configured by cognitive agent and technological digital environment, through possibilities of action that are meaningful in relation to a cognitive agent endowed with skills, bodily capacities, intentions and experiential histories that manifest themselves in patterns of coordinated actions at different levels. This meta-relationship is flexible, dynamic and expansive, therefore, the technological niche grows in relation to the cognitive agent's experience. For example, in the case we address here is corresponding to finger gnosis, we assume that changes in the position and movements of the fingers make it possible to bring a world of numbers and not to have or objectively grasp a world of numbers as is characteristic of representationalist cognitivism. Below we present a brief description of the key concepts that nourish our proposal and that we have borrowed from other articles:

(i) Attentional anchors: during the flow of sensorimotor contingencies of cognitive activity, dynamic equilibrium is instantiated from the attentional anchors that interpolate between the internal dynamics of the agent and the environment in which it participates facilitating emergent understanding (Hutto, Kirkcoff, & Abrahamson, 2015).

(ii) Sensorimotor contingencies: Buhrmann, Di Paolo and Barandiaran (2013) distinguish four types of (SMC) that contribute to specialization of action: (a) sensorimotor environment: intuitive movements of perceptual exploration, without considering sensory feedback (b) sensorimotor habitat: sensory feedback movements between sensory and motor activity as a function of the agent's internal dynamics (c) sensorimotor coordination: specific action patterns that tend to dynamic control according to task goals (d) sensorimotor strategies: optimal balance of the cognitive agent within a normative framework that solves specialized actions.

(iii) Finger gnosis: Butterworth (1999) proposed that fingers are important for representing numerosity. Sinclair and Pimm (2015) have evidenced in their studies that number sense in general is dependent on finger knowledge. Finger knowledge involves a performative gestural act such as (tapping, swiping, pinching and flicking) to produce numbered objects on a multi-touch screen.

In what follows we present Figure 1 as a representation of our proposal. Here we show how the evolution of the performative gesture of finger gnosis (tapping, swiping, pinching and flicking) is linked to the sensorimotor contingencies (sensorimotor environment, sensorimotor habitat, sensorimotor coordination and sensorimotor strategies), as can be seen in the red and blue figures. The segmented lines indicate the dynamic coupling between the cognitive agent and the environment reaffirming the co-dependence or mutualism of reciprocal constitution. Regarding the figures in white that refer to the subpersonal and interpersonal, allusion is made to the progressive coupling in which the cognitive agent reorganizes and specializes its actions to the extent that there is greater sensory feedback with the tactile screen. The bi-directional arrows related to finger gnosis indicate that the specialization of the finger action is co-dependent on the attentional anchors and the sensorimotor contingencies. As the anchors interpolate between the cognitive agent and the touch screen, the sensory variety decreases as a result of the digital affordances that engage the cognitive agent with increasingly specialized actions that transduce number and its operations.

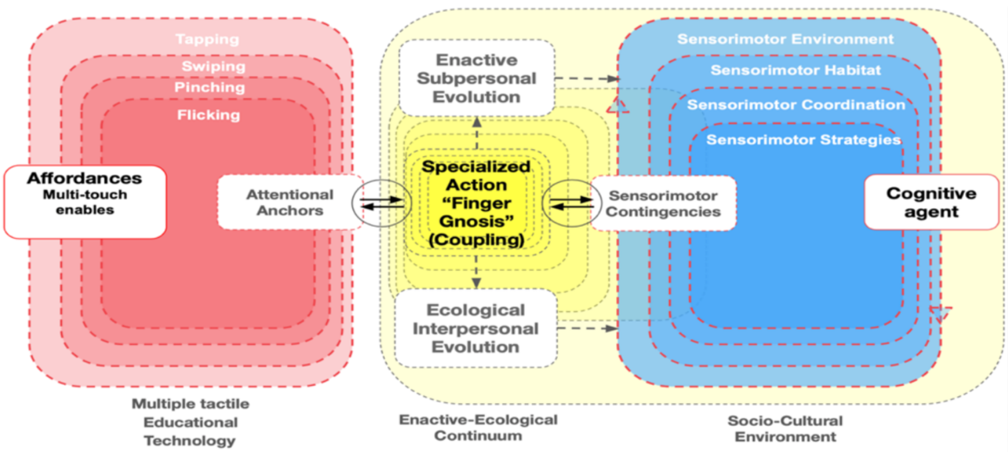
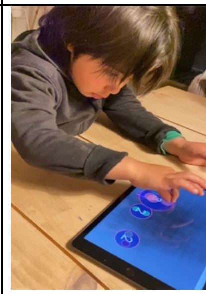
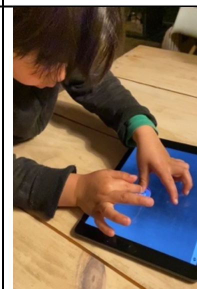
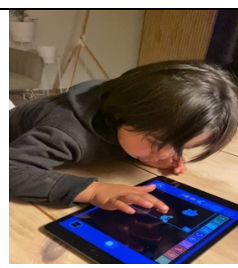


Figure 1: The enactive-ecological continuum model of finger gnosis in numerical cognition with multitouch technology

Empirical findings

To assert our theoretical proposition, we present the excerpt in which author Videla participated in a clinical interview with a 4-year-old kindergarten student named Andy, who interacts for the first time with TouchCounts and Rakin. We chose this excerpt from a larger investigation, as it illustrates a variety of gestures co-dependent on their coupling with the material affordances of the touchscreen. We illustrate that hand and finger movements are becoming specialized in Touchcounts, whereas in Rakin the movements to enact numerical comprehension are restricted. The resulting finger movements have not been explicitly taught, but have emerged in the sensorimotor flow of contingencies in the tactile ecological niche.

Technological ecological niche (Digital affordances)	Finger Gnosis Specialization of tactile action (enacting-ecological numerical comprehension)			
Multiple Tactile	**Tapping (sensoriomotor environment)**	**Swipping (sensoriomotor habitat)**	**Pinching (sensoriomotor coordination)**	**Flicking (sensoriomotor strategies)**
Touchcounts				
Dialogue	**Andy:** I touch and 1 comes out. **Research:** Yes, try again. **Andy**: I play again and it comes out 1, you can also hear 1. **Research:** What else can you do? What appears on the screen? **Andy:** tap and move, numbers come up.	**Andy:** oooh! tap twice and it comes up 2. If I tap three times it comes up 3? **Research:** Yes, and if you tap 4 times how much comes out? **Andy:** 4. Also, you can put the balls together. **Researcher:** Yes, show me. **Andy:** Look, I put two little balls together that say 1 and I get 2.	**Andy:** Now I can form different circles of balls. **Researcher:** How do you do it? **Andy:** I do the little fingers like this (shows the pinching motion by putting thumb and forefinger together on the screen). **Researcher:** When you do those movements, what do you get? Andy: The sum.	**Andy:** You can also separate the balls from the big circle. **Researcher:**Yes, how? Show me. **Andy:** Look, with my two hands I can do it. With one I squeeze and with the other I pull. **Researcher:** What happens to the number in the big circle? Does it increase or decrease? **Andy:** It decreases. **Research:** Why? **Andy:** Because we remove balls.
Rakin				
Dialogue	**Andy:** up come the numbers, I touch 1 and it sounds 1, I touch 2 and it sounds 2. **Research:** Do you know the numbers 1 to 10? **Andy**: Yes. **Research:** What else can you do? What appears on the screen? **Andy**: On the side there are different things that I touch and they sound.	**Andy:** I can touch things and move them here (indicating the central screen divided in two parts). **Research:** Do you see that it changes in the first quadrant? **Andy:** Yes, it changes the amount. **Researcher:** How do you know? What appears on the screen? **Andy:** I move an object and it appears and it sounds 1. I move another one and it sounds 2. **Researcher:** What else can you do with your fingers?		

		Andy: Just move objects to the center.		

Table 1: Specialization of the action of the finger's gnosis in the understanding of the number

Table 1 illustrates the specialization of the action of the knowledge of the fingers for the understanding of the number with the Touchcounts and Rakin applications. This specialization of the action can be seen in the changes presented by the movements of the fingers on the touch screen, of which we have selected (tapping, swiping, pinching and flicking) reported by Sinclair and Pimm (2015) as finger gnosis. In relation to the contrast presented in table 1, we can identify that Andy when participating with Touchcounts begins tapping the multi-touch screen and makes a disc with the number one visually appear and an audio that says one. Later, after several tapping where other discs 1, 2 and 3 appear, he reconfigures this movement for swiping, and where the possibility emerges of making collections of discs that represent new numbers. These two movements correspond to changes in the structural couplings with the touch screen, of which the first are more exploratory and ingenuous where a touch is enough, to move to another sliding movement that makes it possible to join different discs and configure new numbers from the numbers of objects in the popup collection. The reorganisation of these actions is due to the emergence of attentional anchors that interpolate in Andy's internal dynamics and the emerging content of the touch screen. From a numerical understanding, these movements promulgate the notions of ordinality, correspondence, and cardinality. In turn, within the framework of sensorimotor contingencies, it can be established that the tapping and swiping movements in Andy are gestated from the sensorimotor reconfiguration of the environment to the sensorimotor habitat.

This occurs within the framework of changes in the movement of the fingers, the first ingenious and without contact with Andy's internal dynamics (environment) to one with sensory feedback that triggers the sense of agency (habitat). At a phenomenological level of qualitative changes in what we call "finger gnosis", it is possible to observe the alternation of the ring finger with the thumb. Subsequently and in relation to pinching, it can be seen that the movement of the index finger in tapping and the thumb or index finger in swiping, ceases to be one or the other, but transforms into a specialized gesture that encompasses both fingers simultaneously of one hand to respond to emerging objectives. This is relevant, since the effective movement requested by the touch screen is precisely the click to form collections of discs. Andy finds himself moving in some way to bring out new collections in a specialized way. This is what is known from the flow of contingencies in the performative gesture as sensorimotor coordination. Once this movement is coupled to his experience with the pop-up content on the touch screen, he realizes that the collections of discs formed by the pinching movement can also be separated or removed from the collection through a more specialized movement that requires not only sensorimotor coordination of one hand, but it is strategically discovered that you must use both hands to respond effectively to the emerging target. This is known from contingencies as sensorimotor strategies and it is where attentional anchors contribute to sensorimotor ecological control in the understanding of the number: ordinality, correspondence and cardinality.

Regarding the Rakin application, Andy exhibits only the tapping and swiping movements, as evidenced in Table 1. This is not to say that Andy fails to engage in a sustained manner by integrating broader sensorimotor contingencies that contribute to dynamic equilibrium such as sensorimotor coordination and sensorimotor strategies. Rakin offers an environment rich in digital affordances in relation to ordinality, correspondence and cardinality as evidenced by tapping and swiping in which Andy can tap the ordinal sequence of numbers on top and can select from a set of available objects (animals, plants and toys) to establish correspondence and cardinality. Nevertheless, the movements that Rakin requests as digital affordances, do not allow the tactile specialization of the action that does justice to the finger gnosis that leads to enacting numerical comprehension. In this area, we consider that Andy is deployed in a restricted environment of actions that the fingers only lead to tapping and swiping to understand and learn the number. Finally, we consider that Rakin is an important application for the development of the fundamental notions of numerical cognition such as ordinality, correspondence and cardinality, however, it is more framed in a representationalist perspective in which cognition is more linked with representing pre-existing contents that make the contents emerge in the specialization of the action.

Conclusion

In this paper we have considered a proof of concept for enactive and ecological unification in the framework of finger gnosis of number learning with multi-touch technologies such as Touchcounts and Rakin. If our proposal is correct, we contribute to the field of research in embodied perspectives of mathematical cognition that allow to favor instances of learning with technology that resonate with basic forms of actions and gestures.

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